Test Problem #2.

A THREE-DIMENSIONAL RAYLEIGH-TAYLOR MIXING

This problem is similar to problem 3 (Stony Brook conference), proposed by D.Youngs and corresponds approximately to the experimental situation considered by Kucherenko et al. The main purpose of this problem is to compare the results obtained from 3D Direct Numerical Simulation or Large Eddy Simulation using a sub-grid-scale turbulent model. The problem implies mixing two miscible fluids by Rayleigh-Taylor instability.

The fluids are almost incompressible. Hence the problem can be run either on an incompressible or a compressible code.

The experiments were presented at the previous workshop (Kucherenko et al).

The initial geometry

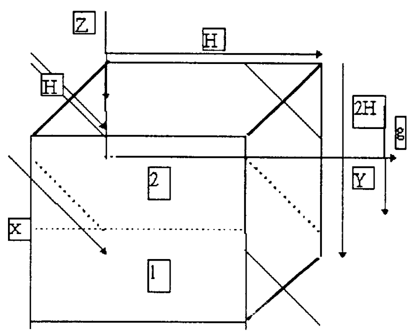

Fig.2. Initial geometry

The computational domain is a cube with H size:

$$Z_1 < Z < Z_2$$
$$Z_1 = -\frac{9}{8}H, \quad Z_2 = \frac{7}{8}H$$
$$0 < X, Y < H.$$

Gravity acts in the z-direction.

For the unperturbed problem:

$$\rho = \begin{cases} \rho 1 & z < 0 \\ \rho 2 & z > 0 \end{cases}$$
$$\frac{\partial p}{\partial z} = \rho g$$

$$P = P_0 \text{ at } Z = 0 .$$

The values chosen for the parameters are:

$$H = 1$$
$$\rho_1 = 1$$
$$\rho_2 = 3$$
$$g = -1$$
$$P_0 = 5$$

Mesh

The problem is chosen so that it can be run on a relatively small mesh (100x100x200). Coarser or finer meshes would also be considered.

Equation of state

If a compressible simulation is used then the equation of state for both fluids is

$$P = (\gamma - 1)\rho e, \ \gamma = 1.4$$

where ρ = density,
e = specific internal energy.

Initial perturbation

Initial perturbation is set as random density distribution of the 1-st substance in one layer of the cells near an unperturbed interface. The distribution formula of density perturbation is

$$\delta \rho = \text{sign}(c-0.5)\rho_1 \delta ,$$

where c is random ($0 < c < 1$), $\delta = 0.1$.

Below the program of generating random c value is appended. When calculating a cycle on cells (at first on axis Y, then on axis X) it is necessary to address this program and to receive the c value. This program will give the same set of random numbers.

For compressible simulations the initial pressure distribution is not changed, i.e, internal energy is defined as:

$$e' = 0.4\, P / \rho'$$

where P = unperturbed pressure
ρ' = perturbed density

For incompressible case the initial pressure distribution should be modified to satisfy the Poisson equation:

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P \right) = 0 \text{ with } \frac{\partial P}{\partial z} = \rho g .$$

Boundary conditions

Reflective boundary conditions are assigned at $z = Z_1, Z_2$. Periodic boundary conditions are assigned in the x and y directions.

Output required

Volume fraction of fluid 2 is used below. If the calculation uses only mass fractions for each fluid (m_r) then the volume fraction for fluid 2 may be defined as

$$f_2 = \frac{m_2 / \rho_2}{m_1 / \rho_1 + m_2 / \rho_2}.$$

Alternatively if the effects of compressibility are neglected we may use

$$f_2 = \frac{\rho - \rho_1}{\rho_1 - \rho_2}.$$

The following output is then proposed. Each item can be conveniently displayed on a single viewgraph.

1. Width of mixing zone L versus $S = gt^2$.
L= distance between the points where $\bar{f}_2 = 0.05$ and $\bar{f}_2 = 0.95$.
The overbar denote the z - plane average.
Suggested size of graph - 7×7 cm.

2. Width of light fluid penetration zone L_1 versus $S = gt^2$.
L_1 is distance between the points where $\bar{f}_2 = 0.95$ and $z = 0$.
Suggested size of graph - 7 ×7 cm.

3. For t = 3; 5.
$\bar{f}_2$ versus z.
Suggested size of graph - 7 ×7 cm.

4. For t=3; 5
3D view of isosurface $f_2 = 0.5$.
Suggested size of each graph - 10 × 10 cm

5. For t = 3; 5 averaged density fluctuation

$$\sigma^2(z) = \overline{(\rho - \bar{\rho})^2}$$

Suggested size of graph - 7 ×7 cm.

6. Maximum value of turbulent energy k_{max} versus t.

$$k_{max} = \max(k(z))$$

$$k = \frac{\overline{(u_i' u_i')}}{2} \quad \text{where} \quad u_i' = u_i - \bar{u}_i.$$